Algebra

1. **What is Algebra?** Algebra is a language. In order to learn algebra, you must learn to read it. Algebra is a system whereby letters called **variables** represent unknown quantities. Algebra uses the same operations as ordinary math such as adding, subtracting, multiplying, and dividing. When using algebra it is **really important** that you understand the 1) basic 4 operations, 2) properties of numbers, 3) Integers, and 4) order of operations (BEDMAS)! Most students who have trouble in algebra also have trouble in one or more of these areas.

2. **Prerequisites to learning algebra**: Before jumping into algebra, let us first look at the four main prerequisites to learning algebra.

   1) **Basic four operations**: If you have find any of these operations difficult you should ask your parents or your teacher to make you up some practice. Algebra will be much easier if you know them very well.

   2) **Properties of numbers**:
      - Special properties of zero. Zero is not positive and not negative. i.e. $3 + 0 = 3$ and $0 + 3 = 3$
        $3 - 0 = 3$ and $0 - 3 = -3$
        $3 \cdot 0 = 0$ and $0 \cdot 3 = 0$
        $0 \div 3 = 0$ BUT $3 \div 0 =$ UNDEFINED
      - The commutative property of addition and multiplication states that the answer (sum or product) will be the same regardless of the order you perform the calculation. i.e. $5+3 = 3+5$ because $8=8$ OR $3 \cdot 5 = 5 \cdot 3$ because $15=15$.

- Associative property of addition and multiplication states that when you add or multiply three numbers together, the answer will be the same no matter how you group them.
  i.e. (3+5) +2 = 3+(2+5) because 8+2 =3+7 (both = 10)
  i.e. (3•2)6 = 3(2•6) because (6)6 = 3(12) (both =36)

- Distributive property of multiplication over addition states that when you multiply a number by an expression in brackets, you must multiply the number in front of the brackets by each of the numbers inside. i.e. 3(5 +2)= 3•5 + 3•2
  because 3(7)=15 +6. (both =21)

- ****These properties only apply to addition and multiplication. They do not work for subtraction or division where the order matters. i.e 7-4 ≠ 4-7.**

3) **Integers:** Made up of all the negative and positive numbers as well as zero. The four operations with integers are important to algebra.

- Adding Integers
  If the *signs are the same*, keep the sign and add the numbers together. -11-6 = -17 OR +12+7 = +19.
  If the *signs are different*, take the sign of the larger number and find the difference in the numbers. -15+12 = -3 OR +12-9=+3

- Subtraction of Integers

  Method 1: Change the sign and the operation. In other words: Add the opposite integer.

  i.e. (-4) - (-4)
  =(-4) + (+4)
  = 0

  i.e. (-6) - (+7)
  =(-6) + (-7)
  = -13

  Method 2: Use the integer rules for multiplication to remove the brackets, then add the numbers.

  i.e. (-4) - (-4)
  = -4 + 4
  = 0

  i.e. (-6) - (+7)
  = -6 - 7
  = -13

  - Multiplication and Division of Integers

    When Signs are different (1 positive, 1 negative), the product or quotient (answer) always ends up with a negative (-) sign.
    i.e (+3)(-4) = -12

    When signs are the same (2 positives or 2 negatives), the product or quotient (answer) always ends up with a positive (+) sign.
    i.e. (-2)(-5) = +10 OR (+3)(+5) =+15

4) **Order of Operations:** Remember to follow order of operations (BEDMAS) when doing algebraic calculations. i.e 36 ÷ (4+8) +2

B	36 ÷ 12 +2
D	3+ 2
A	5

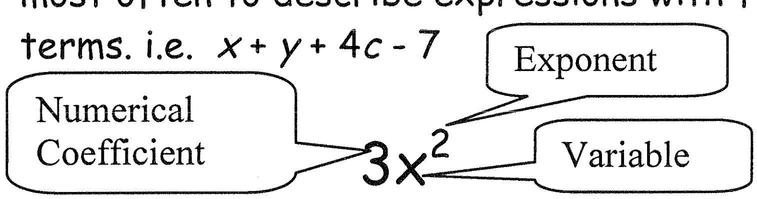
3. **<u>Variables</u>:** The name given to letters that stand for a number. It is also called the **literal part** of a term. Any letter of the alphabet can be used although *a, b, c, n, x, y,* and z are the most common. When you use a letter to stand for a number, you don't know what number the letter is standing for. Think of the letters *x, a, y, z* as mystery numbers. i.e.

$x + 3 = 5$	x is a variable
$a - 2 = 6$	a is a variable
$y \div 3 = 4$	y is a variable
$5z = 10$	z is a variable

Variables can be part of an expression, an equation, or an inequality. A mathematical expression is part of a sentence. Some examples of expressions are: $3x$, x, 5, $x - 2$, $x \div 2$. As you can see, an expression (<u>term</u>) can be a number, a letter, or a combination of a number and a letter. <u>If a term is just a number (integer) it is called a **constant**</u>.

Algebraic Expressions are named based on the number of terms that they have.

- <u>Monomial expressions</u> have one term. i.e. x, 3, -12, $6x$
- <u>Binomial expressions</u> have two unlike terms combined by an addition or subtraction sign. i.e. $x + 3$, $a - 4$, $x - y$
- <u>Trinomial expressions</u> have three unlike terms combined by an addition or subtraction sign. i.e. $2x + 3y - 4$
- <u>Polynomial expressions</u> have one, two, three, or more unlike terms combined by an addition or subtraction sign. This is a general classification for all expressions, used most often to describe expressions with four or more terms. i.e. $x + y + 4c - 7$



<u>***** If the Term is just X the numerical coefficient is 1**</u>

4. **Like/Unlike terms**: Remember a term is a mathematical expression that is either a number, a letter, or a combination of a number and a letter. Like terms have the same variable and the same exponent. You can collect like terms in order to simplify them. If the variables and exponents are the same just add/subtract the **variable's numerical coefficient** (the name given to the number in front of the variable. It tells you how many x's or y's there are. For example in the expression 5x, 5 is the coefficient).

i.e. $\underline{x + 3x} = 4x$

$\underline{6b - 2b} + \underline{5 - 3} = 4b + 2$

$\underline{4x^2 + 3x^2} - 2x^3 + 6 = 7x^2 - 2x^3 + 6$

5. **Operations on algebraic expressions**: Add and subtract as shown above in the collecting terms examples. If they are binomials follow the examples below. See examples of all four operations below.

Adding

$(4x + 5) + (x - 2)$

$= 4x + 5 + x - 2$ — Remove Brackets using integer rules

$= 5x + 3$ — Collect like terms

Subtraction

$(14a - 3) - (2a - 5)$

↓ ↓ — Remove bracket using integer rules, then collect like terms

$= 14a - 3 + -2a + 5$

$= 12a + 2$

Multiplication

$5 \cdot x = 5x$, OR $3(2t - 6) = 6t - 18$

Use distribution: Multiply the 3 by both expressions inside the brackets

Division

$(36n - 6) \div (-3)$

$= -12n + 2$

Divide each expression in the brackets by -3

6. Evaluation of expressions (substitution)

Use if the value or possible values of the variables are known.

A **Domain** of a variable is the set of numbers that could replace a variable and create a true statement. When the algebraic expression is replaced by a number from the domain, it takes the form of a number or numerical value. This means if I tell you that X = 3, then it will be easy to solve X + 7. This would be 3 + 7 OR 10.

i.e Find the value of $x + 2y$ if $x = 2$ and $y = 4$

To Calculate the numerical value of an expression, follow these steps:

1. Write the Algebraic Expression. $x + 2y$
2. Replace each variable with the correct value. $= 2 + 2(4)$
3. Calculate the answer using order of operations. $= 2 + 8$
4. Do calculations vertically, one operation per line. $= 10$

i.e Find the Value of $4a^2 + 2a - 5$ if $a = 3$

1. $4a^2 + 2a - 5$
2. $= 4(3)^2 + 2(3) - 5$
3. $= 4(9) + 2(3) - 5$
4. $= 36 + 6 - 5$

$= 37$

7. **<u>Reading Algebraic expressions:</u>** Look for the following <u>key words</u> and symbols that will help you with the meaning and syntax (order and sign combinations) of algebraic expressions.

Operation	**Algebraic expression**	**Read**
Addition	$x + 5$	x plus five the sum of x and 5 x increased by 5 5 more than x
Subtraction	$x - 5$	x minus five the difference of x *and 5* x diminished by 5 5 less than x
Multiplication	$5x$	five x the product of 5 and x five multiplied by x
Division	$x \div 5$ OR $x / 5$	x over five the quotient of x and 5 x divided by 5 one fifth of x
Exponential	x^2	x squared the square of x x with exponent 2 x to the second power