Algebra

Commutative Property: answer remains the same regardless of the order

Associative Property: answer for 3 numbers remains the same no matter how they are grouped

Distributive Property: must multiply number infront of brackets with each number inside

Algebraic Expressions:

Monomial expression = 1 term
Binomial expression = 2 unlike terms (+ or −)
Trinomial expression = 3 unlike terms (+ or −)
Polynomial expression = 1, 2, 3 or more unlike terms (+ or −)

Example: Trinomial Expression = $2x + 3y + 4$

$3x^2 + 3$ — exponent; constant; variable; numerical coefficient

Domain = A domain of a variable is a set of numbers that could replace a variable and create a true statement. → $x = 3 \rightarrow x + 7 = 3 + 7 = 10$

Examples

$t = -1$ $\quad -2t^2$
$-2(-1)^2$
$-2(1)$
-2

$a = 3$ $\quad 4a^2 + 2a - 5$
$4(3)^2 + 2(3) - 5$
$4(9) + 2(3) - 5$
$36 + 6 - 5$
37

$a = 12$
$b = 1/3$
$c = 5/6$

$$\frac{2a + 12b - 8c - 2(ac)}{4}$$

$12 \times 5 = 60$
$60 \div 6 = 10$
$10 \times 2 = 20$

$$\frac{2(12) + 12(1/3) - 8(5/6) - 2((12)(-5/6))}{4}$$

$$\frac{24 + 4 + 6\,2/3 - 20}{4}$$

$$\frac{54\,2/3}{4}$$

$13\,2/3$

	coefficients	variables	exponents
$-c^2$	−1	c	2
$9x$	9	x	1
y^8	1	y	8
$12a^2b^3$	12	a b	2 3

Repair man earns $3 per hour plus $60 for traveling. Works 15 hours.

$3x + 60$
$3(15) + 60 = 105$

Lunch is half the price of breakfast. Supper is twice the price of lunch. The total is $35.

Total = 35
Breakfast = $\frac{1}{2}x$
Lunch = x
Supper = 2x

$\frac{1}{2}x + x + 2x = 35$
$3.5x = 35$
$3.5 \quad 3.5$
$x = 10$

B = 5
L = 10
S = 20
T = 35

$\frac{x}{2} = 24$
$\times 2 \quad \times 2$
$x = 48$

$\frac{77}{x} = 7$
$\cdot x \quad \cdot x$
$\frac{77}{7} = \frac{7x}{7}$
$11 = x$

$x^2 - 4 = 12$
$+4 \quad +4$
$\sqrt{x^2} = \sqrt{16}$
$x = 4$

$3x - 8 - 2x = -3$
$x - 8 = -3$
$+8 \quad +8$
$x = 5$

$4(x - 3) + 1 = 10 - 3(x - 2)$
$4x - 12 + 1 = 10 - 3x + 6$
$4x - 11 = 16 - 3x$
$+3x \quad +3x$
$7x - 11 = 16$
$+11 \quad +11$
$\frac{7x}{7} = \frac{27}{7}$
$x = 3.85$

$w = 2a^2 - 2a + 50 \quad [a = -10]$
$= 2(-10)^2 - 2(-10) + 50$
$= 2(100) - -20 + 50$
$= 200 + 20 + 50$
$= 270$

$(6x - 3) = (4x - 5)$
$6x - 3 + -4x + 5$
$2x + 2$

Linear Relations

Rate of change = constant of variation = slope

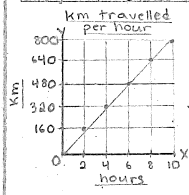
$y = mx + b$
- m: slope
- b: constant
- x: independant variable
- y: dependant variable

x = independant variable
y = dependant variable

unit rate = $y \div x$ $\left(m = \frac{y}{x} \text{ or } \frac{y_2 - y_1}{x_2 - x_1}\right)$

Directly Proportional

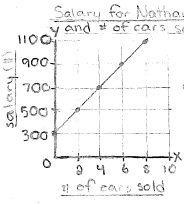
x	0	2	4	6	8	10
y	0	160	320	480	640	800



$y = mx$
$m = \frac{y}{x} = \frac{160}{2} = 80$
$y = 80x$

Indirectly Proportional

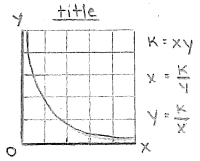
x	0	2	4	6	8	10
y	300	500	700	900	1100	1300



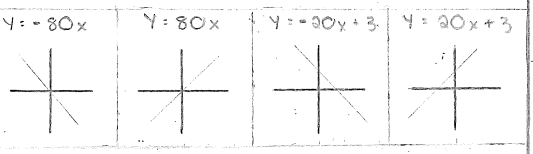
$y = mx + b$
$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{700 - 500}{4 - 2} = \frac{200}{2} = 100$
$y = 100x + b$
$500 = 100(2) + b$
$500 = 200 + b$
$-200 \quad -200$
$300 = b \longrightarrow y = 100x + 300$

Inverse Relation

x	2	4	5	8	10
y	65	32.5	26	16.25	13



$K = xy$
$x = \frac{K}{y}$
$y = \frac{K}{x}$



x-axis = abscissa
y-axis = ordinate

Interest

Simple interest

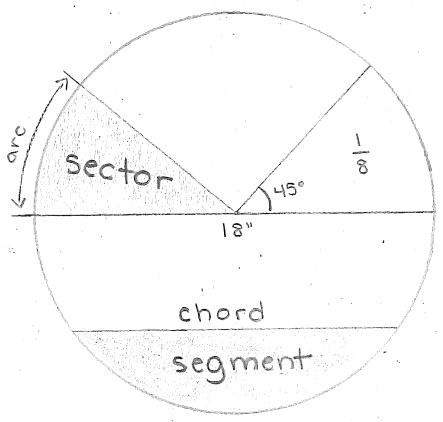
formula: $i = prt$
i = interest earned
p = principal (starting amount)
r = interest rate (as a decimal)
t = time in years

Compound interest

formula: $B = p(1 + r)^t$
B = balance (final amount)
p = principal (starting amount)
r = interest rate (as decimal)
t = time in years

Circles

$c = 2\pi r$ or πd
$d = 2r$ or $\frac{c}{\pi}$
$r = \frac{d}{2}$
$a = \pi r^2$
$p = \frac{2\pi r}{2} + 2r$ (semi-circle)
$p = \frac{\pi d}{2} + d$ (semi-circle)
$p = \frac{2\pi r}{4} + 2r$ (quarter circle)
$p = \frac{\pi d}{4} + d$ (quarter circle)



$d = 18"$
$r = 9"$
$c = \pi d$
$= 3.14 \cdot 18$
$= 56.52"$
$a = \pi r^2$
$= 3.14 \cdot 9^2$
$= 3.14 \cdot 18$
$= 254.34"^2$

$arc = \frac{c}{answer} = \frac{360°}{degrees}$
$= \frac{56.52}{7.065} = \frac{360}{45°}$

or $45 = \frac{1}{8}$ so $\frac{c}{8} = \frac{56.52}{8} = 7.065$

$sector = \frac{a}{answer} = \frac{360°}{degrees}$
$= \frac{254.34}{31.7925} = \frac{360°}{45°}$

Perimeter of $\frac{1}{2}$ the circle:
$p = \frac{2\pi r}{2} + 2r$
$= \frac{3.14 \cdot 9}{2} + 2 \cdot 9$
$= 28.26 + 18$
$= 46.26$

Radius of circle if $p = 46.26$
$p = \frac{2\pi r}{2} + 2r$
$46.26 = \frac{2 \cdot 3.14r}{2} = 2r$
$46.26 = 3.14r + 2r$
$\frac{46.26}{5.14} = \frac{5.14r}{5.14}$
$9 = r$

Geometry

Name	Figure	Area	Perimeter
Triangle	△	$\frac{bh}{2}$	Σs
Parallelogram	▱	bh	Σs
Square	◻	bh	Σs
Rectangle	▭	bh	Σs
Trapezoid	⏢	$\frac{(b_1+b_2)h}{2}$	Σs
Pentagon (+)	⬠	$\frac{Pa}{2}$	Σs
Circle	○	πr^2	$2\pi r$ or πd

Polygon Angles	Formula
sum of interior	$(n-2)180$
sum of exterior	$360°$
each interior	$\frac{(n-2)180}{n}$
each exterior	$\frac{360}{n}$

Name	Figure	Lateral Area	Total Area
Prism	⧈	$P_b \cdot h$	$A_L + A_b \cdot 2$
Cylinder	⌭	$2\pi rh$	$A_L + A_b \cdot 2$
Pyramid	◇	$\frac{P_b \cdot s}{2}$	$A_L + A_b$
Cone	▽	πrs	$A_L + A_b$
Sphere	◯	-	$4\pi r^2$

s = slant or slope

Slant

$h^2 + r^2 = s^2$

height^2 + radius^2 = slant^2

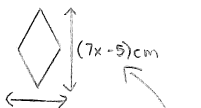
$a^2 + b^2 = c^2$

# of sides	Name	sum of interior angles
3	triangle/trigon	180°
4	quadrilateral	360°
5	pentagon	540°
6	hexagon	720°
7	septagon/heptagon	900°
8	octagon	1080°
9	nonagon	1260°
10	decagon	1440°
11	hendecagon/undecagon	1620°
12	dodecagon	1800°

* multiply/divide by 100 with area or double decimal movement

km	hm	dam	m	dm	cm	mm
0.001	0.01	0.1	1	10	100	1000

÷10 ×10



A = 414cm²

Length of long diagonal?

① $A = \frac{4(7x-5)}{2}$

$= 2(7x-5)$

$= 14x - 10$

② $414 = 14x - 10$

$+10 \quad +10$

$\frac{424}{14} = \frac{14x}{14}$

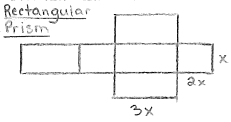
$30.29 = x$

③ $7(30.29) - 5$

$= 212.03 - 5$

$= 207.03$

Rectangular Prism

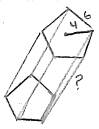


$A_t = P_b \cdot h + A_b \cdot 2$

$= 10x \cdot x + 6x^2 \cdot 2$

$= 10x^2 + 12x^2$

$= 22x^2$



a = 4

side = 6

h = ?

A = 420

$A_t = A_L + A_b \cdot 2$

$420 = P_b \cdot h + \frac{P_a}{2} \cdot 2$

$420 = 5 \cdot 6 \cdot h + 5 \cdot 6 \cdot 4$

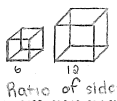
$420 = 30h + 120$

$-120 \quad -120$

$\frac{300}{30} = \frac{30h}{30}$

$10 = h$

Similar Solids



Ratio of sides:

K = 6:12 or 1:2

Ratio of area:

$K^2 = 6^2 : 12^2$

$= 36 : 144$

$= 1 : 4$

or

$K^2 = 1^2 : 2^2$

$= 1 : 4$

Similar Solide

Ratio of sides:

K = 6:18

= 1:3

Ratio of area:

$K^2 = 6^2 : 18^2$

$= 36 : 324$

$= 1 : 9$

$K^3 = 1^3 : 3^3$

$= 1 : 9$

Ratio of area if ratio of sides are:

K = 3:4 ratio of area is 9:16 K^2

K = 5/7 ratio of area is 25.49 K^2

K = 8,2 ratio of area is 16/1 K^2

Ratio of sides if ratio of area is:

$K^2 = 64:49$ ratio of sides is 8/7

$K^2 = 225/484$ ratio of sides is 15/22

$K^2 = 81/144$ ratio of sides is 9/12

$K^2 = 4:41$ ratio of sides is 2/41

Ratio between similar figure: 3/5

Area of similar figure = 81.9cm²

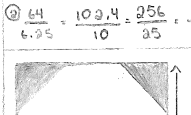
Area of original figure ?

① $3^2 : 9$

$5^2 : 25$

② $\frac{9}{25} = \frac{29.484\text{cm}^2}{81.9}$

Ratio of area of figurine A to C if area of figurine A is 102.4cm²

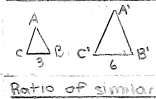


① $8^2 : 64$

$2.5^2 : 6.25$

② $\frac{64}{6.25} = \frac{102.4}{10} = \frac{256}{25} = \frac{64}{6.25}$

Ratio of similarity



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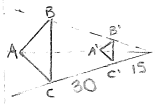
$\frac{6}{3} = \frac{2}{1} = 2$

Ratio of similarity



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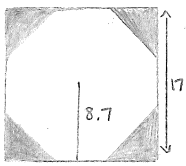
$\frac{15}{45} = \frac{1}{3}$

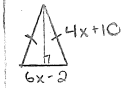


Shaded Area

Square − Octagon

b^2	$\frac{Pa}{2}$
17^2	$\frac{8 \cdot 7 \cdot 8.5}{2}$
289	$\frac{476}{2}$
	238

$289 - 238 = 51\text{m}^2$



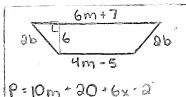
$P = 8x + 20 + 6x - 2$

$= 14x + 18$

$A = \frac{(6x-2)x}{2}$

$= \frac{6x^2 - 2x}{2}$

$= 3x^2 - x$



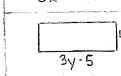
$P = 10m + 20 + 6x \cdot 2$

$A = \frac{(b_1+b_2)h}{2}$

$= \frac{10m + 2(6)}{2}$

$= \frac{60m + 12}{2}$

$= 30m + 6$



$P = 8 + 6y - 10$

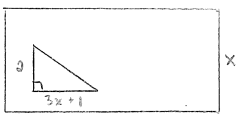
$= 6y - 2$

$A = 4(3y - 5)$

$= 12y - 20$

$A = (2y \cdot 3) + (3 \cdot 8)$

$= 6y + 24$



Shaded Area

① $x(10x - 8)$

$= 10x^2 - 8x$

② $\frac{2(3x+1)}{2}$

$= 3x + 1$

③ $(10x^2 - 8x) - (3x + 1)$

$= 10x^2 - 8x + -3x - 1$

$= 10x^2 - 11x - 1$

Probability

Inapendant: (with replacement)

G	R	Y
4	3	5
$\frac{4}{12}$	$\frac{3}{12}$	$\frac{5}{12}$
$\frac{1}{3}$	$\frac{1}{4}$	$\frac{5}{12}$

$\frac{1}{3} \times \frac{1}{4} \times \frac{5}{12} = \frac{5}{144}$

$P(1G, 1R, 1Y) = \frac{5}{144}$

Dependant: (without replacement)

G	R	Y
4	3	7

$P(1G, 1R, 1Y) =$

$\frac{4}{12} \quad \frac{3}{11} \quad \frac{5}{10}$

$\frac{1}{3} \quad \frac{3}{11} \quad \frac{1}{2}$

$\frac{1}{31} \times \frac{3}{11} \times \frac{1}{2} = \frac{1}{22}$

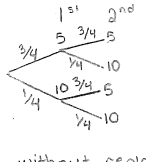
$P(1G, 1R, 1Y) = \frac{1}{22}$

Probability of an outcome:

- with replacement: (2 bills are drawn)

3×$5 1×$10

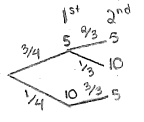
$\Omega = (5,5)(5,10)(10,5)(10,10)$



- without replacement: (2 bills are drawn)

3×$5 1×$10

$\Omega = (5,5)(5,10)(10,5)$



Probability & Statistics

TV watching habits:

Time in minutes	Frequency
0-10	8
10-20	0
20-30	1
30-40	4
40-50	1
50-60	4

Range of TV time:

(largest - smallest)

60-0 = **60**

Mode of TV time:

(occurs most often)

10-10 (highest frequency)

Mean of TV time:

(average)

① find average of each range of numbers

0-10 → 5

10-20 → 15

20-30 → 25

30-40 → 35

40-50 → 45

50-60 → 55

② add them all up

5+15+25+35+45+55=470

③ divide by sum of frequency

sum of frequency:

8+0+1+4+1+4=18

divide:

470 ÷ 18 = **26.1**

Formula of the mean:

$\bar{x} = \frac{\Sigma x}{n}$ mean = $\frac{\text{sum of all value}}{\text{number of values}}$

A pointer is spun twice on the given disc and we use an ordered pair to represent the outcome of this experiment.

Calculate the probability of each outcome

$P(1,1) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

$P(1,3) = \frac{1}{6} \times \frac{1}{3} = \frac{1}{18}$

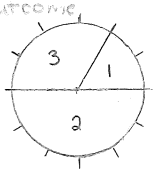
$P(2,3) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{6}$

$P(3,3) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

$P(1,2) = \frac{1}{6} \times \frac{1}{3} = \frac{1}{12}$

$P(2,2) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

$P(3,1) = \frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$



A jar contains 10 balls: 4 = #1, 3 = #2, 2 = #3, 1 = #4

A ball is chosen at random

Calculate the probability of the following events.

A: "Observing an even outcome"

$A = (2,4); P(A) = P(2) + P(4) = 0.4$

B: "Observing an outcome less than 5"

$B = (1,2,3,4) = \Omega; P(B) = P(\Omega) = 1$

C: "Observing an outcome greater than 5"

$C = (\emptyset); P(C) = P(\emptyset) = 0$

A die and a coin are tossed simultaneously. What is the probability of the following events?

The die shows a "6" and the coin shows "tails" 1/12

The die does not show a "6" and the coin shows "tails" 5/12

* The die shows a "6" or the coin shows "tails" $\frac{7}{12} \rightarrow \frac{2}{12} - \frac{1}{12}$

The die shows an even number and the coin shows "tails" 1/4

* The die shows an even number or the coin shows "tails" 3/4

In how many ways can you do this if the 3 boys want to sit beside eachother and the 2 girls want to sit beside eachother?

$(3 \times 2 \times 1) \times (2 \times 1) + (2 \times 1) \times (3 \times 2 \times 1)$

In how many ways can you sit 3 boys and 2 girls if the boys want to sit beside eachother?

$(3 \times 2 \times 1) \times (2 \times 1) + (3 \times 2 \times 1) \times (2 \times 1) + (3 \times 2 \times 1) \times (2 \times 1) = 36$

A die is rolled once. Consider the following events A: "Observing an even outcome", B: "Observing an odd outcome", C: "observing an outcome less than 3", D: "observing an outcome less than 5";

Are the events A and B dependant or independant?

Dependant. When A occurs, B cannot since A and B are incompatible. Thus, the occurence of A influences the probability of B

Are events C and D dependant or independant?

Dependant. Each time C occurs, D also occurs. When an outcome is less than 3, it is automatically less than 5.

Definitions & more Examples

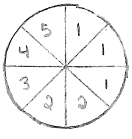
- Ω = omega (set of possibilities)
- equally probable = the set of possibilities Ω is called equally probable space
- probability of an event:

$\Omega = (1,2,3,4,5)$

E: "Observing an even number"

E: (2,4); $P(E) = P(2) + P(4) = \frac{1}{8} + \frac{1}{8} = \frac{2}{8}$

$P(E) = \frac{2}{8}$



- $P(\Omega) = 1$
- $P(\emptyset) = 0$
- $0 \leq P(E) \leq 1$
- probability of an incompatible event = 0
- probability of a complimentary event = 1
- A, B, C = original
- A', B', C' = image (read "A prime, B prime, C prime)
- Dependant and independant events

Two events A and B are independant in probability when the occurence of one event does not influence the probability of the other occuring. On the contrary, the events are called dependant.